# Unit 3: Functions and Patterns



## Unit 3: Functions and Patterns

### Lesson 1: Working with Patterns

Topic 1: Inductive Patterns

*Learning Objectives*

- Recognize patterns in sequences of numbers and geometric/real world objects.
- Apply inductive reasoning to predict the value of unknown terms in a sequence.

Topic 2: Representing Patterns

*Learning Objectives*

- Use tables and graphs to identify and represent patterns.

### Lesson 2: Graphing Functions and Relations

Topic 1: Representing Functions and Relations

*Learning Objectives*

- Define, compare, and recognize relations and functions.
- Represent relations and functions with graphs, tables, and sets of ordered pairs.

Topic 2: Domain and Range

*Learning Objectives*

- Define domain and range.
- Identify the domain and range for relations described with words, symbols, tables, sets of ordered pairs, and graphs.

Topic 3: Proportional Functions

*Learning Objectives*

- Define proportional function.
- Explain the parts of the standard proportional function equation.
- Recognize and describe the characteristics of a proportional function graph.

Topic 4: Linear Functions

*Learning Objectives*

- Define linear functions and describe their characteristics.
- Compare and contrast proportional and non-proportional linear functions.
- Explain the components of the linear function equation.

Topic 5: Non-linear Functions

*Learning Objectives*

- Define nonlinear function.
- Define inverse functions and recognize them in equation, table, and graph form.
- Define quadratic functions and recognize them in equation, table, and graph form.
- Define exponential functions and recognize them in equation, table, and graph form.

## Unit 3 - Media Run Times

**Unit 3**

Lesson 1

Topic 1, Presentation – 4.1 minutes
Topic 1, Worked Example 1 – 2.5 minutes
Topic 1, Worked Example 2 – 6.5 minutes

Topic 2, Presentation – 4.4 minutes
Topic 2, Worked Example 1 – 8.5 minutes
Topic 2, Worked Example 2 – 5.9 minutes

Lesson 2

Topic 1, Presentation – 6.3 minutes
Topic 1, Worked Example 1 – 4.6 minutes
Topic 1, Worked Example 2 – 3.3 minutes

Topic 2, Presentation – 5.1 minutes
Topic 2, Worked Example 1 – 3.5 minutes
Topic 2, Worked Example 2 – 3.1 minutes

Topic 3, Presentation – 4.1 minutes
Topic 3, Worked Example 1 – 3.6 minutes
Topic 3, Worked Example 2 – 4.1 minutes

Topic 4, Presentation – 3.8 minutes
Topic 4, Worked Example 1 – 2.6 minutes
Topic 4, Worked Example 2 – 4.9 minutes

Topic 5, Presentation – 5.5 minutes
Topic 5, Worked Example 1 – 4 minutes
Topic 5, Worked Example 2 – 4.1 minutes

## Unit 3: Functions and Patterns

### Instructor Notes

#### The Mathematics of Functions and Patterns

Unit 3 introduces the idea that the world is full of patterns, patterns that can be found and described mathematically. Students will learn to recognize patterns within and between sequences—in groups of objects, in series of numbers, and in tables and graphs of inputs and outputs. By the time they complete the unit, students will know the definitions and characteristics of proportional, linear, non-linear, inverse, quadratic, and exponential functions.

This unit contains what may appear to be an overwhelming list of objectives:

- Recognize patterns in sequences of numbers and geometrical/real world objects.
- Apply inductive reasoning to predict the value of unknown terms in a sequence.
- Use tables and graphs to identify and represent patterns.
- Define, compare, and recognize relations and functions.
- Represent relations and functions with graphs, tables, and sets of ordered pairs.
- Define domain and range.
- Identify the domain and range for relations described with words, symbols, tables, sets of ordered pairs, and graphs.
- Define proportional function.
- Explain the parts of the standard proportional function equation.
- Recognize and describe the characteristics of a proportional function graph.
- Define linear functions and describe their characteristics.
- Compare and contrast proportional and non-proportional linear functions.
- Explain the components of the linear function equation.
- Define nonlinear function.
- Define inverse functions and recognize them in equation, table, and graph form.
- Define quadratic functions and recognize them in equation, table, and graph form.
- Define exponential functions and recognize them in equation, table, and graph form.

But most of these concepts are only introduced and not explored in depth. For example, the last objective, “define exponential functions and recognize them in equation, table, and graph form” requires that students recognize the general form of an exponential function, but not that they solve one for unknowns. The applications and complexities of functions will generally be left for Algebra 2 and other courses.

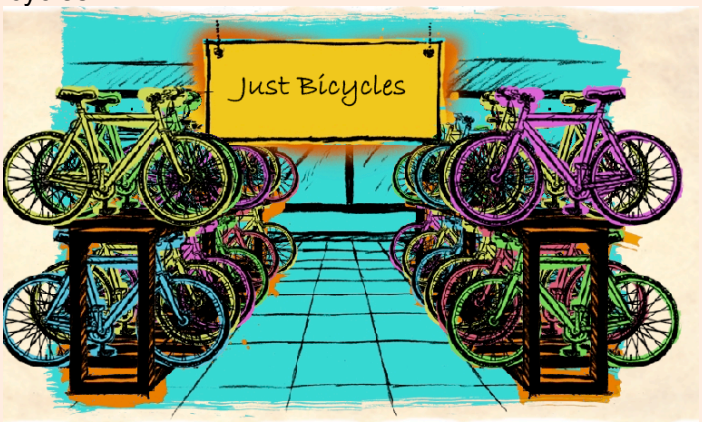
#### Teaching Tips: Conceptual Challenges and Approaches

Students will need to make and use multiple representations of functions and relations, including words, tables, graphs, pictures, and symbolic algebra. The ability to translate

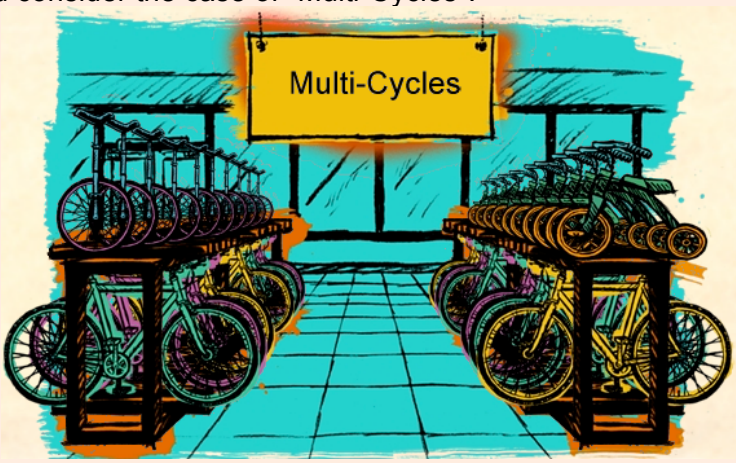
comfortably between these media and understand how to extract information from each context underlies much of a student's potential success in Algebra 1, and beyond.

Allowing students time to generate their own different representations of functions is a critical need. For example, leading students though examples of the typical equation → table → graph relationship is important initially. Once students understand the concept, it is then critical that they have time to develop their own methods for pulling information out of data. This is probably best done in a small group setting, with the teacher facilitating as students explore proportional relationships or patterns.

**Example**
The example used within the video presentation of Lesson 2 Topic 1 illustrates the kind of scenario that works well for this type of semi-independent work. Students could begin with the situation from "Just Bicycles":




Have them create tables, graphs, verbal descriptions, and an equation to describe the relationship between the number of bicycles in the store and the number of wheels.
Then students could consider the case of "Multi-Cycles":




Once again, ask them to develop tables, graphs, verbal descriptions, and an equation (or equations) to describe the relationship between the number of cycles in the store and the number of wheels. When they realize they can't write a simple equation for this relationship, it sets up a discussion of the differences between functions and relations. Many other key topics for the unit, such as domain and range, are also easily developed out of this type of scenario.

***Hands-on Opportunities***

The text of this unit includes 3 manipulatives that lets students to tinker with the graphs of non-linear functions. These are:

- Quadratic Functions (Lesson 2 Topic 5)

  In this manipulative, the values of *a, b,* and *c* in the standard quadratic formula $y = ax^2 + bx + c$ can all be altered. Students will see how the shape and orientation of a parabola is affected by changing these variables.
- Exponential Functions (Lesson 2 Topic 5)

  In this manipulative, the coefficients a and b in the exponential function equation $y = ab^x$ can be changed with slider bars. Students will notice the large effect of small changes on the shape of the graph.

As students create tables and graphs and work with the manipulatives, teachers should ask questions to nudge them into understanding and exploring the meaning of inputs and outputs, relations and functions. Students who spend time discussing these ideas are more likely to understand what they really mean.

**Teaching Tips: Algorithmic Challenges and Approaches**

This unit uses relatively few algorithms, and most of those are very simple, so few students will have trouble with calculations or procedures. However, the ability to accurately draw and interpret graphs is very important. There are aspects of working with graphs that deserve particular focus:

- Students must use rulers to draw graphs and understand that marks on the axes need to be evenly spaced (This can be aided by the use of graph paper in the early stages).

- Appropriate graphs (and/or tables of data) from newspapers or from Internet news articles can be a powerful way of helping students see the connection between the classroom and the world. These examples can also be used to emphasize how reading data incorrectly from a graph might have significant consequences.

- Real-world scenarios can be used to further develop the ideas of function vs. relation, range, domain, and ordered pairs. For example, students could compile the set of people in the class and their heights.

**Summary**

This unit begins the process of helping students visualize and think about numbers, shapes and situations in terms of mathematical patterns. It also introduces the idea of a function, which will be developed more fully in Unit 4: *Analyze and Graph Linear Equations, Functions and Relations* and Unit 10: *Quadratic Functions*.

With lots of hands-on practice creating graphs and charts of inputs and outputs, it is possible to present what at first seems like a daunting list of objectives as a coherent set of interlinked ideas.

**Unit 3: Functions and Patterns**

**Instructor Overview**
**Tutor Simulation: Snowboarding**

## Purpose

This simulation allows students to demonstrate their understanding of functions and relations. Students will be asked to apply what they have learned to solve a real-world problem involving:

- Patterns
- Functions
- Relations
- Domain and range

## Problem

Students are given the following problem:

> We're sending you on a simulated trip to the mountains to go snowboarding. When you get there, you'll notice that the different runs down the mountain--and the lift to the top--all look familiar. Your challenge will be to analyze the different paths up and down the mountain and determine if they are functions and/or relations, linear or non-linear, and proportional or non-proportional.
>
> Next, you'll take a closer look at two of the paths, and use tables and sets of ordered pairs to determine the height to distance rate of change of these paths. Finally you'll develop equations that describe the runs.

## Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- make sure they have completed all other unit material.
- explain the mechanics of tutor simulations
    - Students will be given a problem and then guided through its solution by a video tutor;
    - After each answer is chosen, students should wait for tutor feedback before continuing;
    - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- emphasize that this is an exploration, not an exam.

## Unit 3: Functions and Patterns

### Instructor Overview
### Puzzle: What Comes Next?

#### Objective

*What Comes Next?* is a puzzle that presents a pattern of colored shapes and invites the learner to complete the sequence. Players must recognize the pattern in a series of colored geometric shapes, and apply inductive reasoning to predict the next terms in the sequence.

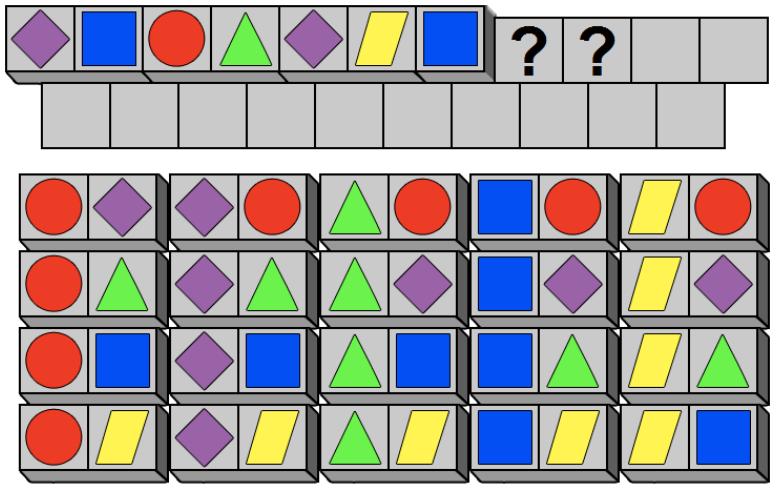



Figure 1. *What Comes Next?* asks players to first determine and then continue the pattern created by a sequence of colored shapes.

#### Description

In each level, players begin with a sequence of colored shapes, and must pick from sets of more shapes to continue the pattern. Learners are rewarded for completing the sequences correctly, and the faster they do it the more lavish the praise. There are three levels of difficulty—the easy level has a 3 shape pattern, the medium level starts with a 5 piece pattern, and the difficult level has 7 shapes in the initial sequence.

There are ten set sequences at each level, so the game play is not endless. However, the more difficult the patterns the less likely the inferences necessary to complete the sequence will be made from memory than afresh. *What Comes Next?* is suitable for solo play, for two or more players, or even in a classroom setting where learners could call out the pairs by color or shape or both.

## Unit 3: Functions and Patterns

### Instructor Overview
### Project: Design a Rollercoaster

**Student Instructions**

**Introduction**
Almost everyone has ridden or at least seen a roller coaster in action. Did you know that there is a connection between roller coasters and the algebra you have been studying in this unit? Engineers rely on their knowledge of algebra to design roller coasters to make them both fun and scary at the same time. Use these websites to help you study the algebra involved in roller coaster designs.

http://www.funderstanding.com/coaster
http://mathdl.maa.org/images/upload_library/4/vol5/coaster/coasterapplet.htm
http://nlvm.usu.edu/en/nav/frames_asid_331_g_3_t_2.html?from=category_g_3_t_2.html

**Task**
For this project you or your team will use your knowledge of algebra functions and patterns to determine how engineers use their knowledge of math to design roller coasters. You will draw a picture of roller coaster and plot ordered pairs along the path a coaster car travels on the coaster's rails. You will also compare these ordered points by constructing a functions table and a graph to plot the ordered points. You can use the websites to help explore the design of roller coasters, identifying ordered points on a coaster, and review how to plot ordered points on a graph.

**Instructions**
Draw the side view of a roller coaster of your choice or one of your own design on a piece of poster board. The roller coaster should have at least 2 hills and 1 loop. Layout both the "X" and "Y" axis, labeling each axis with appropriate intervals for selecting ordered pairs. Select ordered pairs, one pair at interval points of your choice on the drawing along the track of your roller coaster. Record these ordered points in a function table and then create a graph of these values.

Problem 1:

Write a 1 – 2 page paper that compares and contrasts how your roller coaster design relates to proportional function. Include a graph and table to support your discussion.

Problem 2:

Write a 1 – 2 page paper that compares and contrasts how your roller coaster design relates to linear or non-linear function. Include a graph and table to support your discussion.

Problem 3:

Create an interactive poster using Glogster to explain why or why not your roller coaster has a relationship between its design and functions in algebra.

**Collaboration**
Discuss with other students or team members the relationship between your roller coaster and the design of other roller coasters. Use pictures of roller coasters at Hershey Park or Six Flags Great America for comparison at the following websites:

http://www.coastergallery.com/2000T/hp.html
http://www.coastergallery.com/1999T/SFGA.html

**Conclusions**
Share the findings of your project with others using any of the following methods:

- Present the answers to the problems and interactive poster to the other students in your class.
- Present the answers to the problems and interactive poster to the other students in your class using a multimedia presentation.
- Post the answers on the class Wiki for sharing with parents and fellow students. Request feedback and comments from those view your work.

## Instructor Notes

### Assignment Procedures

**Problem 1**
This task focuses on use of sets of ordered pairs (taken from the roller coaster design) to recognize a proportional function. Instruct students that their description should include a discussion of whether their roller coaster is proportional or non-proportional, along with appropriate formula(s) to support their findings.

**Problem 2**
This question asks students to recognize and compare the linear and non-linear portions of their roller coaster. Instruct them to explain their reasoning, along with the appropriate formula(s) to support their findings.

**Problem 3**
This question requires students to make an interactive poster of their findings in earlier problems. Look up Glogster (http://edu.glogster.com) as a class and make sure everyone understands how to find and use the site.

Suggest that students incorporate video clips to illustrate their project. Videos of roller coasters are available on YouTube, or students can create one at http://www.animoto.com/education.

Recommendations:

- have students work in teams to encourage brainstorming and cooperative learning.
- assign a specific timeline for completion of the project that includes milestone dates.

- provide students feedback as they complete each milestone.
- ensure that each member of student groups has a specific job.

**Technology Integration**

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology. The following are examples of free internet resources that can be used to support this project:

http://www.moodle.org
An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview Lets you create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work with parents.

http://www.docs.google.com
Allows students to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/
The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

**Rubric**

| Score | Content | Presentation |
|---|---|---|
| **4** | Your project includes a complete understanding of the algebra functions required to solve the problems.<br><br>Your written reports use appropriate algebra functions to answer problems.<br><br>Your project uses appropriate graphs and tables to support written reports.<br><br>Your interactive poster includes all the appropriate pictures, video(s), graphs, and tables. | Your presentation is detailed and clear. Explanation includes the important algebra functions required in the project in logical sequence that is easy to understand. |
| **3** | Your project includes a good understanding of the algebra functions required to solve | Your presentation is clear. Explanation includes the important |

| | | |
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| | the problems.<br><br>Your written reports use appropriate algebra functions to answer problems.<br><br>Your project uses appropriate graphs and tables to support written reports.<br><br>Your poster includes all the appropriate pictures, video(s), graphs, and tables. However, it is not interactive. | algebra functions required in the project in logical sequence that is easy to understand. |
| **2** | Your project includes a good understanding of the algebra functions required to solve the problems.<br><br>Your written reports do not include all the appropriate algebra functions to answer problems.<br><br>Your project's graphs and tables to support written reports.<br><br>Your poster pictures, video(s), graphs, and tables do not support the algebra ideas of this unit. However, it is interactive. | Your presentation is a little difficult to understand. However, it does include most of the important algebra functions required in the project. |
| **1** | Your project includes does not show you understand the algebra functions required to solve the problems.<br><br>Your written reports does is indicate you understand all the appropriate algebra functions to answer problems.<br><br>Your project's written reports do not include graphs and tables.<br><br>You did not complete the interactive poster. | Your presentation is difficult to understand and does not follow a logical thought process. Several important algebra functions required in the project are missing. |

## Unit 3: Algebra - Functions and Patterns

### Glossary

| | |
|---|---|
| **constant of proportionality** | the constant in a proportional function equation; it describes the ratio or proportional relationship of the independent and dependent variables—also called the constant of variation or the rate of change |
| **constant of variation** | the constant in a proportional function equation; it describes the ratio or proportional relationship of the independent and dependent variables—also called the rate of change or the constant of proportionality |
| **continuous pattern** | a pattern made of uninterrupted or connected values or objects |
| **dependent value** | a value or variable that depends upon the independent value |
| **dependent variable** | a value or variable that depends upon the independent value |
| **discrete pattern** | a pattern made of separate and distinct values or objects |
| **discrete values** | values that change in increments (not continuously) |
| **domain** | the set of all possible inputs of a function which allow the function to work |
| **exponential function** | a nonlinear function in which the independent value is an exponent in the function, as in y = abx |
| **function** | a kind of relation in which one variable uniquely determines the value of another variable |
| **independent value** | a value or variable that changes or can be manipulated by circumstances |
| **independent variable** | a value or variable that changes or can be manipulated by circumstances |
| **inductive reasoning** | a form of logical thinking that makes general conclusions based on specific situations, inductive reasoning takes the path of observation to generalization to conjecture |
| **input** | the independent variable of a function—input determines output |
| **inverse function** | a nonlinear function in which the reciprocal of the independent variable times a constant equals the dependent variable, as in $y = \frac{k}{x}$ |
| **linear function** | a function with a constant rate of change and a straight line graph |
| **mathematical sequence** | an ordered list of numbers or objects |

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| **range** | the set of all possible outputs of a function |
| **rate of change** | the constant in a proportional function equation; it describes the ratio or proportional relationship of the independent and dependent variables—also called the constant of variation or the constant of proportionality |
| **relation** | the relationship between variables that change together |
| **term** | a value in a sequence--the first value in a sequence is the 1st term, the second value is the 2nd term, and so on; a term is also any of the monomials that make up a polynomial |

# NROC Algebra 1--An Open Course
# Unit 3
# Mapped to Common Core State Standards, Mathematics

Algebra 1 | Functions and Patterns | Working with Patterns | Inductive Patterns

**Grade: 7 - Adopted 2010**

| | | |
|---|---|---|
| **STRAND / DOMAIN** | **CC.7.MP.** | Mathematical Practices |
| **CATEGORY / CLUSTER** | **7.MP.2.** | Reason abstractly and quantitatively. |
| **CATEGORY / CLUSTER** | **7.MP.7.** | Look for and make use of structure. |
| **CATEGORY / CLUSTER** | **7.MP.8.** | Look for and express regularity in repeated reasoning. |

**Grade: 8 - Adopted 2010**

| | | |
|---|---|---|
| **STRAND / DOMAIN** | **CC.8.MP.** | Mathematical Practices |
| **CATEGORY / CLUSTER** | **8.MP.2.** | Reason abstractly and quantitatively. |
| **CATEGORY / CLUSTER** | **8.MP.7.** | Look for and make use of structure. |
| **CATEGORY / CLUSTER** | **8.MP.8.** | Look for and express regularity in repeated reasoning. |

**Grade: 9-12 - Adopted 2010**

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|---|---|---|
| **STRAND / DOMAIN** | **CC.MP.** | Mathematical Practices |
| **CATEGORY / CLUSTER** | **MP-2.** | Reason abstractly and quantitatively. |
| **CATEGORY / CLUSTER** | **MP-7.** | Look for and make use of structure. |
| **CATEGORY / CLUSTER** | **MP-8.** | Look for and express regularity in repeated reasoning. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Understand the concept of a function and use function notation. |
| **EXPECTATION** | **F-IF.3.** | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n greater than or equal to 1. |

Algebra 1 | Functions and Patterns | Working with Patterns | Representing Patterns

**Grade: 9-12 - Adopted 2010**

| | | |
|---|---|---|
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Understand the concept of a function and use function notation. |
| **EXPECTATION** | **F-IF.3.** | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n greater than or equal to 1. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-BF.** | Building Functions |
| **STANDARD** | | Build a function that models a relationship between two quantities. |
| **EXPECTATION** | **F-BF.1.** | Write a function that describes a relationship between two quantities. |
| **GRADE EXPECTATION** | **F-BF.1.a.** | Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |

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| CATEGORY / CLUSTER | F-BF. | Building Functions |
| STANDARD | | Build a function that models a relationship between two quantities. |
| EXPECTATION | F-BF.2. | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |

Algebra 1 | Functions and Patterns | Graphing Functions and Relations | Representing Functions and Relations

**Grade: 7 - Adopted 2010**

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| STRAND / DOMAIN | CC.7.MP. | Mathematical Practices |
| CATEGORY / CLUSTER | 7.MP.4. | Model with mathematics. |

**Grade: 8 - Adopted 2010**

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| STRAND / DOMAIN | CC.8.MP. | Mathematical Practices |
| CATEGORY / CLUSTER | 8.MP.4. | Model with mathematics. |
| STRAND / DOMAIN | CC.8.EE. | Expressions and Equations |
| CATEGORY / CLUSTER | | Understand the connections between proportional relationships, lines, and linear equations. |
| STANDARD | 8.EE.5. | Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. |
| STRAND / DOMAIN | CC.8.F. | Functions |
| CATEGORY / CLUSTER | | Define, evaluate, and compare functions. |
| STANDARD | 8.F.1. | Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. |

**Grade: 9-12 - Adopted 2010**

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| STRAND / DOMAIN | CC.MP. | Mathematical Practices |
| CATEGORY / CLUSTER | MP-4. | Model with mathematics. |
| STRAND / DOMAIN | CC.F. | Functions |
| CATEGORY / CLUSTER | F-IF. | Interpreting Functions |
| STANDARD | | Understand the concept of a function and use function notation. |
| EXPECTATION | F-IF.1. | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). |
| STRAND / DOMAIN | CC.F. | Functions |
| CATEGORY / CLUSTER | F-IF. | Interpreting Functions |
| STANDARD | | Analyze functions using different representations. |
| EXPECTATION | F-IF.9. | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| STRAND / DOMAIN | CC.M. | Modeling |
| CATEGORY / CLUSTER | M-2. | Formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables |

Algebra 1 | Functions and Patterns | Graphing Functions and Relations | Domain and Range

**Grade: 7 - Adopted 2010**

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|---|---|---|
| **STRAND / DOMAIN** | **CC.7.MP.** | Mathematical Practices |
| **CATEGORY / CLUSTER** | **7.MP.4.** | Model with mathematics. |

**Grade: 8 - Adopted 2010**

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|---|---|---|
| **STRAND / DOMAIN** | **CC.8.MP.** | Mathematical Practices |
| **CATEGORY / CLUSTER** | **8.MP.4.** | Model with mathematics. |
| **STRAND / DOMAIN** | **CC.8.EE.** | Expressions and Equations |
| **CATEGORY / CLUSTER** | | Understand the connections between proportional relationships, lines, and linear equations. |
| **STANDARD** | **8.EE.5.** | Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. |

**Grade: 9-12 - Adopted 2010**

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| **STRAND / DOMAIN** | **CC.MP.** | Mathematical Practices |
| **CATEGORY / CLUSTER** | **MP-4.** | Model with mathematics. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Interpret functions that arise in applications in terms of the context. |
| **EXPECTATION** | **F-IF.5.** | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Analyze functions using different representations. |
| **EXPECTATION** | **F-IF.9.** | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| **STRAND / DOMAIN** | **CC.M.** | Modeling |
| **CATEGORY / CLUSTER** | **M-2.** | Formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables |

Algebra 1 | Functions and Patterns | Graphing Functions and Relations | Proportional Functions

**Grade: 7 - Adopted 2010**

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| **STRAND / DOMAIN** | **CC.7.RP.** | Ratios and Proportional Relationships |
| **CATEGORY / CLUSTER** | | Analyze proportional relationships and use them to solve real-world and mathematical problems. |
| **STANDARD** | **7.RP.2.** | Recognize and represent proportional relationships between quantities. |
| **EXPECTATION** | **7.RP.2.a.** | Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. |
| **EXPECTATION** | **7.RP.2.c.** | Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. |

**Grade: 8 - Adopted 2010**

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| **STRAND / DOMAIN** | **CC.8.EE.** | Expressions and Equations |
| **CATEGORY / CLUSTER** | | Understand the connections between proportional relationships, lines, and linear equations. |
| **STANDARD** | **8.EE.5.** | Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. |
| **STRAND / DOMAIN** | **CC.8.F.** | Functions |
| **CATEGORY / CLUSTER** | | Use functions to model relationships between quantities. |
| **STANDARD** | **8.F.4.** | Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. |

**Grade: 9-12 - Adopted 2010**

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| **STRAND / DOMAIN** | **CC.A.** | Algebra |
| **CATEGORY / CLUSTER** | **A-CED.** | Creating Equations |
| **STANDARD** | | Create equations that describe numbers or relationships. |
| **EXPECTATION** | **A-CED.1.** | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. |
| **EXPECTATION** | **A-CED.2.** | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. |
| **EXPECTATION** | **A-CED.3.** | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. |
| **STRAND / DOMAIN** | **CC.A.** | Algebra |
| **CATEGORY / CLUSTER** | **A-REI.** | Reasoning with Equations and Inequalities |
| **STANDARD** | | Understand solving equations as a process of reasoning and explain the reasoning. |
| **EXPECTATION** | **A-REI.1.** | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |
| **STRAND / DOMAIN** | **CC.A.** | Algebra |
| **CATEGORY / CLUSTER** | **A-REI.** | Reasoning with Equations and Inequalities |
| **STANDARD** | | Represent and solve equations and inequalities graphically. |
| **EXPECTATION** | **A-REI.10.** | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). |

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| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Interpret functions that arise in applications in terms of the context. |
| **EXPECTATION** | **F-IF.4.** | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |

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| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Analyze functions using different representations. |
| **EXPECTATION** | **F-IF.7.** | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| **GRADE EXPECTATION** | **F-IF.7.a.** | Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Analyze functions using different representations. |
| **EXPECTATION** | **F-IF.9.** | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-BF.** | Building Functions |
| **STANDARD** | | Build a function that models a relationship between two quantities. |
| **EXPECTATION** | **F-BF.1.** | Write a function that describes a relationship between two quantities. |
| **GRADE EXPECTATION** | **F-BF.1.a.** | Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-BF.** | Building Functions |
| **STANDARD** | | Build new functions from existing functions. |
| **EXPECTATION** | **F-BF.4.** | Find inverse functions. |
| **GRADE EXPECTATION** | **F-BF.4.a.** | Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-LE.** | Linear and Exponential Models |
| **STANDARD** | | Construct and compare linear and exponential models and solve problems. |
| **EXPECTATION** | **F-LE.1.** | Distinguish between situations that can be modeled with linear functions and with exponential functions. |
| **GRADE EXPECTATION** | **F-LE.1.a.** | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-LE.** | Linear and Exponential Models |
| **STANDARD** | | Construct and compare linear and exponential models and solve problems. |
| **EXPECTATION** | **F-LE.2.** | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |

Algebra 1 | Functions and Patterns | Graphing Functions and Relations | Linear Functions

**Grade: 7 - Adopted 2010**

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| **STRAND / DOMAIN** | **CC.7.RP.** | Ratios and Proportional Relationships |
| **CATEGORY / CLUSTER** | | Analyze proportional relationships and use them to solve real-world and mathematical problems. |
| **STANDARD** | **7.RP.2.** | Recognize and represent proportional relationships between quantities. |

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| EXPECTATION | 7.RP.2.a. | Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. |

**Grade: 8 - Adopted 2010**

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| STRAND / DOMAIN | CC.8.EE. | Expressions and Equations |
| CATEGORY / CLUSTER | | Understand the connections between proportional relationships, lines, and linear equations. |
| STANDARD | 8.EE.5. | Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. |
| STRAND / DOMAIN | CC.8.EE. | Expressions and Equations |
| CATEGORY / CLUSTER | | Analyze and solve linear equations and pairs of simultaneous linear equations. |
| STANDARD | 8.EE.8. | Analyze and solve pairs of simultaneous linear equations. |
| EXPECTATION | 8.EE.8.a. | Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. |
| EXPECTATION | 8.EE.8.b. | Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. |

**Grade: 9-12 - Adopted 2010**

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| STRAND / DOMAIN | CC.A. | Algebra |
| CATEGORY / CLUSTER | A-CED. | Creating Equations |
| STANDARD | | Create equations that describe numbers or relationships. |
| EXPECTATION | A-CED.2. | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. |
| STRAND / DOMAIN | CC.A. | Algebra |
| CATEGORY / CLUSTER | A-REI. | Reasoning with Equations and Inequalities |
| STANDARD | | Solve systems of equations. |
| EXPECTATION | A-REI.6. | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. |
| STRAND / DOMAIN | CC.A. | Algebra |
| CATEGORY / CLUSTER | A-REI. | Reasoning with Equations and Inequalities |
| STANDARD | | Represent and solve equations and inequalities graphically. |
| EXPECTATION | A-REI.10. | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). |
| EXPECTATION | A-REI.11. | Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. |
| STRAND / DOMAIN | CC.F. | Functions |
| CATEGORY / CLUSTER | F-IF. | Interpreting Functions |
| STANDARD | | Interpret functions that arise in applications in terms of the context. |

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| **EXPECTATION** | **F-IF.4.** | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Analyze functions using different representations. |
| **EXPECTATION** | **F-IF.7.** | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| **GRADE EXPECTATION** | **F-IF.7.a.** | Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-LE.** | Linear and Exponential Models |
| **STANDARD** | | Construct and compare linear and exponential models and solve problems. |
| **EXPECTATION** | **F-LE.1.** | Distinguish between situations that can be modeled with linear functions and with exponential functions. |
| **GRADE EXPECTATION** | **F-LE.1.a.** | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. |

Algebra 1 | Functions and Patterns | Graphing Functions and Relations | Non-linear Functions

**Grade: 8 - Adopted 2010**

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| **STRAND / DOMAIN** | **CC.8.F.** | Functions |
| **CATEGORY / CLUSTER** | | Define, evaluate, and compare functions. |
| **STANDARD** | **8.F.2.** | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. |
| **STANDARD** | **8.F.3.** | Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. |
| **STRAND / DOMAIN** | **CC.8.F.** | Functions |
| **CATEGORY / CLUSTER** | | Use functions to model relationships between quantities. |
| **STANDARD** | **8.F.5.** | Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. |

**Grade: 9-12 - Adopted 2010**

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| **STRAND / DOMAIN** | **CC.A.** | Algebra |
| **CATEGORY / CLUSTER** | **A-SSE.** | Seeing Structure in Expressions |
| **STANDARD** | | Write expressions in equivalent forms to solve problems. |
| **EXPECTATION** | **A-SSE.3.** | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. |
| **GRADE EXPECTATION** | **A-SSE.3.c.** | Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as (1.15^(1/12))^12t approximately equals 1.012^12t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. |
| **STRAND / DOMAIN** | **CC.A.** | Algebra |

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| **CATEGORY / CLUSTER** | **A-CED.** | Creating Equations |
| **STANDARD** | | Create equations that describe numbers or relationships. |
| **EXPECTATION** | **A-CED.2.** | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. |
| **STRAND / DOMAIN** | **CC.A.** | Algebra |
| **CATEGORY / CLUSTER** | **A-REI.** | Reasoning with Equations and Inequalities |
| **STANDARD** | | Represent and solve equations and inequalities graphically. |
| **EXPECTATION** | **A-REI.10.** | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Interpret functions that arise in applications in terms of the context. |
| **EXPECTATION** | **F-IF.4.** | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Analyze functions using different representations. |
| **EXPECTATION** | **F-IF.7.** | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| **GRADE EXPECTATION** | **F-IF.7.a.** | Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| **GRADE EXPECTATION** | **F-IF.7.e.** | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-IF.** | Interpreting Functions |
| **STANDARD** | | Analyze functions using different representations. |
| **EXPECTATION** | **F-IF.8.** | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. |
| **GRADE EXPECTATION** | **F-IF.8.b.** | Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)^12t, y = (1.2)^t/10, and classify them as representing exponential growth or decay. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |
| **CATEGORY / CLUSTER** | **F-LE.** | Linear and Exponential Models |
| **STANDARD** | | Construct and compare linear and exponential models and solve problems. |
| **EXPECTATION** | **F-LE.1.** | Distinguish between situations that can be modeled with linear functions and with exponential functions. |
| **GRADE EXPECTATION** | **F-LE.1.a.** | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. |
| **GRADE EXPECTATION** | **F-LE.1.b.** | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. |
| **GRADE EXPECTATION** | **F-LE.1.c.** | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. |
| **STRAND / DOMAIN** | **CC.F.** | Functions |

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| **CATEGORY / CLUSTER** | **F-LE.** | Linear and Exponential Models |
| **STANDARD** | | Construct and compare linear and exponential models and solve problems. |
| **EXPECTATION** | **F-LE.3.** | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |